

AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the application:

Listing of Claims:

- 1 1. (Currently amended) A method for using a computer system to solve a
2 global inequality constrained optimization problem specified by a function f and a
3 set of inequality constraints $p_i(\mathbf{x}) \leq 0$ ($i=1, \dots, m$), wherein f and p_i are scalar
4 functions of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$, the method comprising:
5 receiving a representation of the function f and the set of inequality
6 constraints at the computer system;
7 storing the representation in a memory within the computer system;
8 performing an interval inequality constrained global optimization process
9 to compute guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$
10 subject to the set of inequality constraints;
11 wherein performing the interval global optimization process involves,
12 applying term consistency to the set of inequality
13 constraints over a sub-box \mathbf{X} , and
14 excluding any portion of the sub-box \mathbf{X} that is proved to be
15 in violation of at least one member of the set of inequality
16 constraints; and
17 recording the guaranteed bounds in the computer system memory;
18 wherein applying term consistency involves:
19 symbolically manipulating an equation within the computer
20 system to solve for a term, $g(x'_j)$, thereby producing a modified

21 equation $g(x'_j) = h(\mathbf{x})$, wherein the term $g(x'_j)$ can be analytically
 22 inverted to produce an inverse function $g^{-1}(\mathbf{y})$;
 23 substituting the sub-box \mathbf{X} into the modified equation to
 24 produce the equation $g(\mathbf{X}'_j) = h(\mathbf{X})$;
 25 solving for $\mathbf{X}'_j = g^{-1}(h(\mathbf{X}))$; and
 26 intersecting \mathbf{X}'_j with the j -th element of the sub-box \mathbf{X} to
 27 produce a new sub-box \mathbf{X}^+ ;
 28 wherein the new sub-box \mathbf{X}^+ contains all solutions of the
 29 equation within the sub-box \mathbf{X} , and wherein the size of the new
 30 sub-box \mathbf{X}^+ is less than or equal to the size of the sub-box \mathbf{X} .

1 2. (Previously presented) The method of claim 1, further comprising:
 2 linearizing the set of inequality constraints to produce a set of linear
 3 inequality constraints with interval coefficients that enclose the nonlinear
 4 constraints;
 5 preconditioning the set of linear inequality constraints through additive
 6 linear combinations to produce a preconditioned set of linear inequality
 7 constraints;
 8 applying term consistency to the set of preconditioned linear inequality
 9 constraints over the sub-box \mathbf{X} , and
 10 excluding any portion of the sub-box \mathbf{X} that violates any member of the set
 11 of preconditioned linear inequality constraints.

1 3. (Original) The method of claim 2, further comprising:
 2 keeping track of a least upper bound f_bar of the function $f(\mathbf{x})$ at a feasible
 3 point \mathbf{x} wherein $p_i(\mathbf{x}) \leq 0$ ($i=1, \dots, m$); and
 4 including $f(\mathbf{x}) \leq f_bar$ in the set of inequality constraints prior to
 5 linearizing the set of inequality constraints.

1 4. (Original) The method of claim 2, further comprising removing from
2 consideration any inequality constraints that are not violated by more than a
3 specified amount for purposes of applying term consistency prior to linearizing
4 the set of inequality constraints.

1 5. (Previously presented) The method of claim 1, wherein performing the
2 interval global optimization process involves:
3 keeping track of a least upper bound f_bar of the function $f(\mathbf{x})$ at a feasible
4 point \mathbf{x} ;
5 removing from consideration any sub-box for which $f(\mathbf{x}) > f_bar$;
6 applying term consistency to the f_bar inequality $f(\mathbf{x}) \leq f_bar$ over the sub-
7 box \mathbf{X} ; and
8 excluding any portion of the sub-box \mathbf{X} that violates the f_bar inequality.

1 6. (Previously presented) The method of claim 1, wherein if the sub-box \mathbf{X}
2 is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$), performing the interval global
3 optimization process involves:
4 determining a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes
5 components $g_i(\mathbf{x})$ ($i=1, \dots, n$);
6 removing from consideration any sub-box for which $\mathbf{g}(\mathbf{x})$ is bounded away
7 from zero, thereby indicating that the sub-box does not include an extremum of
8 $f(\mathbf{x})$; and
9 applying term consistency to each component $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$
10 over the sub-box \mathbf{X} ; and
11 excluding any portion of the sub-box \mathbf{X} that violates any component of
12 $\mathbf{g}(\mathbf{x})=\mathbf{0}$.

1 7. (Previously presented) The method of claim 1, wherein if the sub-box **X**
2 is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$), performing the interval global
3 optimization process involves:
4 determining diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the Hessian of the
5 function $f(\mathbf{x})$;
6 removing from consideration any sub-box for which $H_{ii}(\mathbf{x})$ a diagonal
7 element of the Hessian over the sub-box **X** is always negative, indicating that the
8 function f is not convex over the sub-box **X** and consequently does not contain a
9 global minimum within the sub-box **X**;
10 applying term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the
11 sub-box **X**; and
12 excluding any portion of the sub-box **X** that violates a Hessian inequality.

1 8. (Previously presented) The method of claim 1, wherein if the sub-box **X**
2 is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$), performing the interval global
3 optimization process involves:
4 performing the Newton method, wherein performing the Newton method
5 involves,
6 computing the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the gradient of the
7 function f evaluated with respect to a point \mathbf{x} over the sub-box **X**,
8 computing an approximate inverse **B** of the center of
9 $\mathbf{J}(\mathbf{x}, \mathbf{X})$,
10 using the approximate inverse **B** to analytically determine
11 the system $\mathbf{B}\mathbf{g}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$,
12 and wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$);
13 applying term consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for
14 each variable x_i ($i=1, \dots, n$) over the sub-box **X**; and

15 excluding any portion of the sub-box **X** that violates a component.

1 9 (Canceled).

1 10. (Original) The method of claim 1, further comprising performing the
2 Newton method on the John conditions.

1 11. (Currently amended) A computer-readable storage medium storing
2 instructions that when executed by a computer cause the computer to perform a
3 method for using a computer system to solve a global inequality constrained
4 optimization problem specified by a function f and a set of inequality constraints
5 $p_i(\mathbf{x}) \leq 0$ ($i=1, \dots, m$), wherein f is a scalar function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$,
6 the method comprising:
7 receiving a representation of the function f and the set of inequality
8 constraints at the computer system;
9 storing the representation in a memory within the computer system;
10 performing an interval inequality constrained global optimization process
11 to compute guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$
12 subject to the set of inequality constraints;
13 wherein performing the interval global optimization process involves,
14 applying term consistency to the set of inequality
15 constraints over a sub-box **X**, and
16 excluding any portion of the sub-box **X** that is proved to be
17 in violation of at least one member of the set of inequality
18 constraints; and
19 recording the guaranteed bounds in the computer system memory;
20 wherein applying term consistency involves:

21 symbolically manipulating an equation within the computer
 22 system to solve for a term, $g(x'_j)$, thereby producing a modified
 23 equation $g(x'_j) = h(\mathbf{x})$, wherein the term $g(x'_j)$ can be analytically
 24 inverted to produce an inverse function $g^{-1}(\mathbf{y})$;
 25 substituting the sub-box \mathbf{X} into the modified equation to
 26 produce the equation $g(\mathbf{X}'_j) = h(\mathbf{X})$;
 27 solving for $\mathbf{X}'_j = g^{-1}(h(\mathbf{X}))$; and
 28 intersecting \mathbf{X}'_j with the j -th element of the sub-box \mathbf{X} to
 29 produce a new sub-box \mathbf{X}^+ ;
 30 wherein the new sub-box \mathbf{X}^+ contains all solutions of the
 31 equation within the sub-box \mathbf{X} , and wherein the size of the new
 32 sub-box \mathbf{X}^+ is less than or equal to the size of the sub-box \mathbf{X} .

1 12. (Previously presented) The computer-readable storage medium of
 2 claim 11, wherein the method further comprises:
 3 linearizing the set of inequality constraints to produce a set of linear
 4 inequality constraints with interval coefficients that enclose the nonlinear
 5 constraints;
 6 preconditioning the set of linear inequality constraints through additive
 7 linear combinations to produce a preconditioned set of linear inequality
 8 constraints;
 9 applying term consistency to the set of preconditioned linear inequality
 10 constraints over the sub-box \mathbf{X} , and
 11 excluding any portion of the sub-box \mathbf{X} that violates any member of the set
 12 of preconditioned linear inequality constraints.

1 13. (Original) The computer-readable storage medium of claim 12,
 2 wherein the method further comprises:

3 keeping track of a least upper bound f_bar of the function $f(\mathbf{x})$ at a feasible
4 point \mathbf{x} wherein $p_i(\mathbf{x}) \leq 0$ ($i=1, \dots, m$); and
5 including $f(\mathbf{x}) \leq f_bar$ in the set of inequality constraints prior to
6 linearizing the set of inequality constraints.

1 14. (Original) The computer-readable storage medium of claim 12,
2 wherein the method further comprises removing from consideration any inequality
3 constraints that are not violated by more than a specified amount for purposes of
4 applying term consistency prior to linearizing the set of inequality constraints.

1 15. (Previously presented) The computer-readable storage medium of
2 claim 11, wherein performing the interval global optimization process involves:
3 keeping track of a least upper bound f_bar of the function $f(\mathbf{x})$ at a feasible
4 point \mathbf{x} ;
5 removing from consideration any sub-box for which $f(\mathbf{x}) > f_bar$;
6 applying term consistency to the f_bar inequality $f(\mathbf{x}) \leq f_bar$ over the sub-
7 box \mathbf{X} ; and
8 excluding any portion of the sub-box \mathbf{X} that violates the f_bar inequality.

1 16. (Previously presented) The computer-readable storage medium of
2 claim 11, wherein if the sub-box \mathbf{X} is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$),
3 performing the interval global optimization process involves:
4 determining a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes
5 components $g_i(\mathbf{x})$ ($i=1, \dots, n$);
6 removing from consideration any sub-box for which $\mathbf{g}(\mathbf{x})$ is bounded away
7 from zero, thereby indicating that the sub-box does not include an extremum of
8 $f(\mathbf{x})$; and

9 applying term consistency to each component $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$
10 over the sub-box \mathbf{X} ; and
11 excluding any portion of the sub-box \mathbf{X} that violates any component of
12 $\mathbf{g}(\mathbf{x})=\mathbf{0}$.

1 17. (Previously presented) The computer-readable storage medium of
2 claim 11, wherein if the sub-box \mathbf{X} is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$),
3 performing the interval global optimization process involves:
4 determining diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the Hessian of the
5 function $f(\mathbf{x})$;
6 removing from consideration any sub-box for which $H_{ii}(\mathbf{x})$ a diagonal
7 element of the Hessian over the sub-box \mathbf{X} is always negative, indicating that the
8 function f is not convex over the sub-box \mathbf{X} and consequently does not contain a
9 global minimum within the sub-box \mathbf{X} ;
10 applying term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the
11 sub-box \mathbf{X} ; and
12 excluding any portion of the sub-box \mathbf{X} that violates a Hessian inequality.

1 18. (Previously presented) The computer-readable storage medium of
2 claim 11, wherein if the sub-box \mathbf{X} is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$),
3 performing the interval global optimization process involves:
4 performing the Newton method, wherein performing the Newton method
5 involves,
6 computing the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the gradient of the
7 function f evaluated with respect to a point \mathbf{x} over the sub-box \mathbf{X} ,
8 computing an approximate inverse \mathbf{B} of the center of
9 $\mathbf{J}(\mathbf{x}, \mathbf{X})$,

10 using the approximate inverse \mathbf{B} to analytically determine
11 the system $\mathbf{B}\mathbf{g}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$,
12 and wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$);
13 applying term consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for
14 each variable x_i ($i=1, \dots, n$) over the sub-box \mathbf{X} ; and
15 excluding any portion of the sub-box \mathbf{X} that violates a component.

1 19 (Canceled).

1 20. (Original) The computer-readable storage medium of claim 11,
2 wherein the method further comprises performing the Newton method on the John
3 conditions.

1 21. (Currently amended) An apparatus for using a computer system to
2 solve a global inequality constrained optimization problem specified by a function
3 f and a set of inequality constraints $p_i(\mathbf{x}) \leq 0$ ($i=1, \dots, m$), wherein f is a scalar
4 function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$, the apparatus comprising:
5 a receiving mechanism that is configured to receive a representation of the
6 function f and the set of inequality constraints at the computer system;
7 a memory within the computer system for storing the representation;
8 a global optimizer that is configured to perform an interval inequality
9 constrained global optimization process to compute guaranteed bounds on a
10 globally minimum value of the function $f(\mathbf{x})$ subject to the set of inequality
11 constraints;
12 a term consistency mechanism within the global optimizer that is
13 configured to,
14 apply term consistency to the set of inequality constraints
15 over a sub-box \mathbf{X} , and to

16 exclude any portion of the sub-box \mathbf{X} that is proved to be in
17 violation of at least one member of the set of inequality constraints;
18 and
19 a recording mechanism that is configured record the guaranteed bounds in
20 the computer system memory;
21 wherein the term consistency mechanism is configured to:
22 symbolically manipulate an equation within the computer
23 system to solve for a term, $g(x'_j)$, thereby producing a modified
24 equation $g(x'_j) = h(\mathbf{x})$, wherein the term $g(x'_j)$ can be analytically
25 inverted to produce an inverse function $g^{-1}(\mathbf{y})$;
26 substitute the sub-box \mathbf{X} into the modified equation to
27 produce the equation $g(\mathbf{X}'_j) = h(\mathbf{X})$;
28 solve for $\mathbf{X}'_j = g^{-1}(h(\mathbf{X}))$; and
29 intersect \mathbf{X}'_j with the j -th element of the sub-box \mathbf{X} to
30 produce a new sub-box \mathbf{X}^+ ;
31 wherein the new sub-box \mathbf{X}^+ contains all solutions of the
32 equation within the sub-box \mathbf{X} , and wherein the size of the new
33 sub-box \mathbf{X}^+ is less than or equal to the size of the sub-box \mathbf{X} .

1 22. (Previously presented) The apparatus of claim 21, further comprising:
2 a linearizing mechanism that is configured to linearize the set of inequality
3 constraints to produce a set of linear inequality constraints with interval
4 coefficients that enclose the nonlinear constraints; and
5 a preconditioning mechanism that is configured to precondition the set of
6 linear inequality constraints through additive linear combinations to produce a
7 preconditioned set of linear inequality constraints;
8 wherein the term consistency mechanism is configured to,

9 apply term consistency to the set of preconditioned linear
10 inequality constraints over the sub-box **X**, and to
11 exclude any portion of the sub-box **X** that violates any
12 member of the set of preconditioned linear inequality constraints.

1 23. (Original) The apparatus of claim 22, wherein the global optimizer is
2 configured to:

3 keep track of a least upper bound f_bar of the function $f(\mathbf{x})$ at a feasible
4 point \mathbf{x} wherein $p_i(\mathbf{x}) \leq 0$ ($i=1, \dots, m$); and to
5 include $f(\mathbf{x}) \leq f_bar$ in the set of inequality constraints prior to linearizing
6 the set of inequality constraints.

1 24. (Original) The apparatus of claim 22, wherein the term consistency
2 mechanism is configured to remove from consideration any inequality constraints
3 that are not violated by more than a specified amount for purposes of applying
4 term consistency prior to linearizing the set of inequality constraints.

1 25. (Previously presented) The apparatus of claim 21,
2 wherein the global optimizer is configured to,
3 keep track of a least upper bound f_bar of the function $f(\mathbf{x})$
4 at a feasible point \mathbf{x} , and to
5 remove from consideration any sub-box for which
6 $f(\mathbf{x}) > f_bar$;
7 wherein the term consistency mechanism is configured to,
8 apply term consistency to the f_bar
9 inequality $f(\mathbf{x}) \leq f_bar$ over the sub-box **X**, and to
10 exclude any portion of the sub-box **X** that
11 violates the f_bar inequality.

1 26. (Previously presented) The apparatus of claim 21, wherein if the sub-
2 box \mathbf{X} is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$):
3 the global optimizer is configured to,
4 determine a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$
5 includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$), and to
6 remove from consideration any sub-box for which $\mathbf{g}(\mathbf{x})$ is
7 bounded away from zero, thereby indicating that the sub-box does
8 not include an extremum of $f(\mathbf{x})$; and
9 the term consistency mechanism is configured to,
10 apply term consistency to each component $g_i(\mathbf{x})=0$
11 ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=0$ over the sub-box \mathbf{X} , and to
12 exclude any portion of the sub-box \mathbf{X} that violates any
13 component of $\mathbf{g}(\mathbf{x})=0$.

1 27. (Previously presented) The apparatus of claim 21, wherein if the sub-
2 box \mathbf{X} is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$):
3 the global optimizer is configured to,
4 determine diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the
5 Hessian of the function $f(\mathbf{x})$, and to
6 remove from consideration any sub-box for which $H_{ii}(\mathbf{x})$ a
7 diagonal element of the Hessian over the sub-box \mathbf{X} is always
8 negative, indicating that the function f is not convex over the sub-
9 box \mathbf{X} and consequently does not contain a global minimum within
10 the sub-box \mathbf{X} ; and
11 the term consistency mechanism is configured to,
12 apply term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$
13 ($i=1, \dots, n$) over the sub-box \mathbf{X} , and to

14 exclude any portion of the sub-box **X** that violates a
15 Hessian inequality.

1 28. (Previously presented) The apparatus of claim 21, wherein if the sub-
2 box **X** is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$):
3 the global optimizer is configured to perform the Newton method, wherein
4 performing the Newton method involves,
5 computing the Jacobian **J**(**x**,**X**) of the gradient of the
6 function f evaluated with respect to a point **x** over the sub-box **X**,
7 computing an approximate inverse **B** of the center of
8 **J**(**x**,**X**), and
9 using the approximate inverse **B** to analytically determine
10 the system **Bg**(**x**), wherein **g**(**x**) is the gradient of the function $f(\mathbf{x})$,
11 and wherein **g**(**x**) includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$); and
12 the term consistency mechanism is configured to,
13 apply term consistency to each component $(\mathbf{Bg}(\mathbf{x}))_i = 0$
14 ($i=1, \dots, n$) for each variable x_i ($i=1, \dots, n$) over the sub-box **X**, and to
15 exclude any portion of the sub-box **X** that violates a
16 component.

1 29 (Canceled).

1 30. (Original) The apparatus of claim 21, wherein the global optimizer is
2 configured to apply the Newton method to the John conditions.